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Classical electrodynamics with non-point charge: big computational difficulties generated by small parameters

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Abstract. We describe computational difficulties that may arise in the numerical solution of the stationary problem of classical electrodynamics with non-point particles [1]. These difficulties are associated with the presence of a small parameter in this theory, which takes into account the curvature of spacetime caused by the existence of a non-point charged particle. The energy of a gravitational coupling almost completely “eats"the electromagnetic energy of a particle. To calculate the observed particle mass, it is necessary to calculate these two opposite-sign contributions to the particle mass with an accuracy greater than 21 significant digits. Such unprecedented requirements for accuracy did not arise earlier in any problem of theoretical physics.

Keywords: non-point leptons, Stoney’s mass, electromagnetic mass, gravitational coupling energy.

1. Introduction

The author’s article [1] presented a new formulation of classical electrodynamics, which does not contain point charges. The non-point charges of this theory are some eigenstates of the current field J^i. The 4-current J^i (more precisely, J^i is the density of 4-current) inside the particle is a space-like vector: $J^iJ_i < 0$. The motivation for the space-likeness of the current J^i is presented in [1]. The condition of the space-like current means that in the theory [1] the 4-current J^i has no mechanical interpretation. The space part $\mathbf{J}$ of the 4-current J^i cannot be interpreted in terms of a motion, a space-transfer of a charge density ρ. Within the framework of this theory, the 4-current $J^i = \{\rho c, \mathbf{J}\}$ is the primary physical object, whose properties cannot be expressed in terms of the known properties of some other, simpler physical objects.

Within the framework of the theory [1], the electromagnetic field inside the particle is described by a vector dyad consisting of two 4-vectors: 4-current J^i and 4-potential A^i. Inside the particle, the current J^i is a space-like vector. On the outer boundary of a non-point particle, the 4-current J^i is an isotropic 4-vector: $J^iJ_i = 0$. This boundary condition of the 4-current isotropy implicitly determines the outer boundary of the particle. On this boundary, the derivatives of the 4-potential A^i are continuous. The components A^i on the surface of a particle can undergo a discontinuity. Outside of charges, the current J^i is absent, and the 4-potential A^i satisfies the homogeneous Maxwell equations. In the framework of the classical theory [1], the 4-current J^i is taken into account in the Lagrangian density by the term L_J quadratic in the current:

$L_J = -\frac{1}{2}aJ^iJ_i$. For dimensional reasons, the constant factor a in L_J can be written as $a = \left(\frac{r_0}{c}\right)^2$, where c is the speed of light, and r_0 is the new fundamental constant of length dimension. The constant r_0 is some estimation of the size of non-point particles. It is well known that, up to distances of $\sim 10^{-16}$ cm, massive leptons do not demonstrate the presence of any internal structure in particle scattering experiments. Therefore, $r_0 < 10^{-16}$ cm. Below it is shown that the real particle size is much smaller than this experimental upper limit. Small particle sizes[1] mean a high density of the components of the energy-momentum tensor of the currents and the electromagnetic field T^{ij} inside the charge and in its vicinity. This means that classical electrodynamics of non-point particles requires taking into account the curvature of space-time inside and around the particles [1]. Consequently, the electrodynamics equations of non-point particles must be solved in a Riemannian space-time with geometry which obeys the Einstein equations and is unknown before the solution. When calculating the mass and angular momentum of a lepton, it is necessary to take into account not only the contribution of the tensor T^{ij}. It is necessary to take into account the contribution made to these integral characteristics by the pseudo-tensor of the energy-momentum of the gravitational field t^{ij}. This need to take into account the contribution t^{ij} to the integral characteristics of particles gives rise to the computational problem of such a difficulty, which, apparently, no one in the computational mathematics has ever faced before.

2. Dimensionless parameters of classical electrodynamics of non-point particles

The system of equations of classical electrodynamics of the non-point participles [1] includes the Einstein equation. Consequently, the theory contains a gravitational constant G. This means that along with the unknown fundamental constant r_0, the theory contains another constant of the dimension of length. This is the so-called "the Stoney's length" [3] $r_s : r_s = \frac{e}{c^2}\sqrt{G} = 1.381 \cdot 10^{-34}$ cm, where e is the value of the electric charge of an electron. In the article [1] it was suggested that the value of r_s characterizes the dimensions of some current-free internal cavities inside the particles.

The theory [1] also contains some characteristic quantity having the dimension of mass, so-called "the Stoney's mass" $m_s : m_s = \frac{e}{\sqrt{G}} = 1.859 \cdot 10^{-6}$ g[2]. Note that "the Stoney's units" r_s and m_s appeared in theoretical physics in 1881, i.e. earlier than

[1] We are talking only about massive leptons. The study of hadrons in the framework of the approach [1] requires the construction of the classical theory of the Yang-Mills octuplet field, consisting of the octuplet dyad: the 4-current octuplet and the 4-potential octuplet. The author's article [2] gives a general idea of a such theory. Quarks in theory [2] are one-current extended objects immersed in a triplet or a quadruplet of potentials that form a subset of the potential octuplet that is closed relative to the vector product in a vector octuplet. This vector product is specified by means of the structure anti-symmetric 3-symbol of the group $SU(3)$. This combination of three or four potentials forms what is commonly called the color of the quark. There are 8 quarks in theory [2], two of which are not yet open.

[2] The British-Irish physicist and astronomer George Johnstone Stoney not only introduced the fundamental system of units of measurement of physical quantities, but also proposed in 1874 a new

the Planck units that are more common for modern physicists: the Planck mass is $m_p = \sqrt{\frac{\hbar c}{G}} = 2.176 \cdot 10^{-5}$ g and the Planck length is $r_p = \sqrt{\frac{G\hbar}{c^3}} = 1.616 \cdot 10^{-33}$ cm ($\hbar$ is Planck's constant). We believe that in the classical theory [1], the appearance of Planck's constant and, accordingly, of Planck units of length and mass is completely inappropriate.

The presence in the theory [1] of two constants with dimension of a length means that the classical electrodynamics of non-point particles "at the entrance" to the theory contains the dimensionless parameter $\kappa = \frac{r_s}{r_0}$. The presence of this parameter "at the entrance" to the theory – it is very reasonable to assume that it is a small parameter – makes it possible to calculate and explain some small parameters appearing "at the output" of the theory. The theory [1] allows, in principle, to calculate the masses of charged leptons, expressing them, for example, through the Stoney's mass. However, such a dimensionless electron mass μ is monstrously small: $\mu = \frac{m}{m_s} = 0.49 \cdot 10^{-21}$ (here m is the electron mass in grams).

The author provided an estimate of the possible value of the fundamental constant $r_0 : r_0 \approx \left(r_T {r_s}^2\right)^{1/3} \simeq 2 \cdot 10^{-27}$ cm [5], where $r_T = \frac{e^2}{mc^2}$ is the Thompson radius of the electron[3].

It is unlikely that this estimate of the constant r_0 is very reliable, because r_0 is a fundamental constant that characterizes the structure of the world as a whole, and r_T is a constant that characterizes only one particle (electron). But there is currently no other estimation for r_0. From this estimation for r_0 it follows that $\kappa \simeq \left(\frac{m}{m_s}\right)^{1/3} \simeq 0.8 \cdot 10^{-7}$ and, accordingly, $\mu \simeq \kappa^3$. These estimations for r_0 and κ are only by orders of magnitude. The relationship between μ and κ may contain a factor of $O(1)$.

This fundamental relation $\mu \simeq \kappa^3$ is based on the qualitative and not very reliable arguments given in [5]. Generally speaking, it should be an exact quantitative consequence of the numerical solution of the stationary problem about geometry, charge and current distribution for classical model of a non-point lepton [1]. However, this stationary problem has not yet been solved, and the difficulties of its numerical solution now seem insurmountable.

3. Small parameters of the theory [1] and high demands on the accuracy of the numerical solution

Without having the solution of the stationary single-particle problem of classical electrodynamics of non-point particles, we can present here only qualitative considerations illustrating the difficulty of numerically solving this problem. To estimate the observed lepton mass m, we can offer the following formula:

$$m = m_{em} - \frac{U_G}{c^2}, \tag{3.1}$$

term "electron" as the name of an elementary particle that was not yet open at that time ([4], p. 82).

[3] In formula (39) of article [4] in this expression, instead of the Stoney's length r_s, the Plank's length r_p appeared. The author believes now that the appearance of Planck units is inappropriate in the classical theory.

where m_{em} is the electromagnetic mass (it is the integral taken over the entire three-dimensional space in the particle rest system from the component T^{00} of the energy-momentum tensor of the current and the electromagnetic field). The value $-\ U_G$ is the energy of the gravitational coupling (it is the integral taken over the entire three-dimensional space in the particle rest system from the component t^{00} of the pseudo-tensor of the energy-momentum of the gravitational field).

Electromagnetic mass of the particle m_{em} for dimensional reasons can be written in the following form:

$$m_{em} = A\frac{e^2}{r_0 c^2}, \tag{3.2}$$

where A is some dimensionless constant. It can be assumed that $A \gg 1$, since the contribution to m_{em} is made not only by the Coulomb energy (the integral of ρ^2), but also by the interaction energy of the currents (the integral of $\mathbf{J}^2$). By virtue of the space-likeness of the 4-current $\mathbf{J}^2 \geq \rho^2 c^2$. In addition, the values of ρ^2 and $\mathbf{J}^2$ can increase with the deepening from the outer surface into the depth of the charge.

The energy of the gravitational coupling U_G can be estimated by the following "Newtonian" formula:

$$U_G = BG\frac{m_{em}^2}{r_0}, \tag{3.3}$$

where B is some dimensionless constant. We venture to assume that $B \gg 1$ and, moreover, we venture to assume that the factors A and B have the same order of magnitude.

Using formulas (3.1), (3.2) and (3.3) to for an estimation the observed lepton mass m, one can get the following expression:

$$m = m_s \cdot A\kappa\mathcal{R}, \tag{3.4}$$

where $\mathcal{R}$ is the "reducing factor";

$$\mathcal{R} = 1 - AB\kappa^2. \tag{3.5}$$

Earlier we found that for an electron in order of magnitude:

$$\mu \simeq \frac{m}{m_s} \simeq \kappa^3.$$

Therefore, from (3.4) it follows that

$$A\kappa\mathcal{R} \simeq \kappa^3. \tag{3.6}$$

Taking into account (3.5), condition (3.6) can be satisfied if we assume that the unknown factors A and B have order порядок κ^{-1}:

$$A = \frac{a}{\kappa}, \qquad B = \frac{b}{\kappa}, \tag{3.7}$$

$$(a = O(1), \quad b = O(1)) \, ,$$

moreover, the values of a and b, appearing in (3.7), are related by a rigid relation:

$$\mathcal{R} = 1 - ab = \lambda\kappa^3. \tag{3.8}$$

$$(\lambda = O(1)) \, .$$

Equation (3.8), connecting a and b, looks extremely unattractive. It resembles the well-known “fine-tuning” problem [6].

The term “fine-tuning” is used for the high-precision adjustment of some parameters of a physical theory (in this case, parameters a and b) in order to reconcile the theory with observations.

However, relations (3.7) and (3.8) only by appearance remind of the “fine-tuning” problem. Parameters A and B are not external parameters, the value of which can be specified arbitrarily. They should be found in the numerical solution of the stationary single-particle problem of classical electrodynamics with non-point leptons, described in [1]. This numerical solution, if and when it will be obtained, either demonstrates the validity of the estimation of the reducing factor $\mathcal{R}$ (3.8) – and then the theory [1] agrees with the observed facts (at least qualitatively) – or does not confirm the estimate (3.8), – and then the theory [1] should be rejected.

If theory [1] is suitable for describing experimental facts, then relation (3.8) demonstrates the existence of a giant computational problem. Electromagnetic energy and the energy of a gravitational coupling compensate each other with an accuracy of 20 significant digits. On the Stoney's mass scale, leptons are practically massless[4]. In order to obtain meaningful results when calculating the mass and angular momentum of a charged particle, these integral characteristics should be calculated, providing at least 21 reliable significant figures. The ratio of the muon mass to the electron mass is known with an accuracy of 7 significant digits. In order to reproduce this accuracy in calculations, the integral characteristics of two solutions with different topology must be calculated with an accuracy of 28 significant digits. Accordingly, when numerically solving this problem, for example, using the grid method, calculating local values of variables (the potential and current components, the electromagnetic field tensor, the metric tensor components and the Christoffel symbols), it is necessary to produce with an adequate margin of accuracy (probably at least 24-25 significant digits for comparison of integral characteristics with experience in order of magnitude, or with an accuracy of 31-32 significant figures to ensure accuracy in integral characteristics comparable to the experimental one). At present, there are no such exact numerical methods for solving a system of nonlinear partial differential equations in domains with unknown boundaries. In the past, not a single problem of theoretical physics and continuum mechanics put such high demands on the accuracy of calculations. However,

[4]Here we are faced with the “the inverted hierarchy problem” [7] the gravitational interaction is not at all extremely weak compared to other types of interaction (as is customary to think about it now). On the contrary, gravity ***almost*** completely “devours” the energy contributions of other types of interaction. The world is ***almost*** massless.

at present, there is no other classical theory, except [1], which would allow, in principle, to hope for the calculation of the masses of leptons[5]. Relation (3.8) can be checked by performing model calculations for given and not very small κ, for example, $\kappa \sim 0.1$ and $\kappa \sim 0.01$. Of course, any attempt to solve this problem numerically may be preceded by an attempt to prove a theorem on the existence of solutions to this problem. Currently, such evidence is missing.

The considered problem is a very rare example of the problem of theoretical physics, which requires proof of a theorem on the existence of a solution. Physicists usually do not have a need for theorems of this kind. In the scientific folklore of theoretical physics on this subject, the mocking phrase of the ruthless W. Pauli is known, allegedly pronounced in the 40s of the twentieth century to the famous mathematician John von Neumann: "Wenn Physik hauptsächlich aus Beweisen bestehen würde, wäre von Neumann ein guter Physiker geworden" (If physics consisted mainly of proofs, von Neumann would have become a good Physicist)[6].

However, this is exactly the case when such a theorem is important. It is unlikely that anyone will decide to tackle the numerical solution of such a problem with unprecedentedly high demands on accuracy, without having confidence in the existence of a solution. Possible classical solutions to this problem may differ from each other in the topology of the external and internal boundaries of the current and charge distribution in the particle volume. These topologically different solutions (if they exist) are classical models of various massive leptons. If we confine ourselves to axisymmetric solutions, then there are only three solutions: the outer boundary with the topology of the sphere is compatible with the existence of the inner boundary with the topology of the sphere or the topology of the torus; the outer boundary with the topology of the torus admits the existence of an inner boundary with the topology of the torus. By now, three massive leptons are really known: an electron, a muon and a triton.

4. Questions that can be answered by the theorem on the existence of solutions for the theory [1]

Reducing the equations of classical electrodynamics with non-point charge [1] to a dimensionless form, it is convenient to choose the values of e, c and r_0 as units of measurement. The equations of electrodynamics themselves in such a dimensionless record do not contain any parameters[7]. The dimensionless constant of the theory appears in the Einstein equation:

$$\mathcal{R}_{ik} - \frac{1}{2} g_{ik} \mathcal{R} = Gr \cdot T_{ik} \tag{4.1}$$

[5] These monstrous computational difficulties are the price that must be paid for the abandonment of the existing physics with ***point*** particles, divergences and renormalizations.

[6] Unfortunately, the author cannot indicate the source of this phrase. This is probably one of the many jokes about Pauli.

[7] The theory [1], in addition to Maxwell's equations, contains one new and very simple equation of theoretical physics, relating 4-current and 4-potential in the 4-current region: $J^i + A^i = 0$.

where $Gr = 8\pi\kappa^2$. Gr is the dimensionless Einstein gravitational constant expressed in terms of the parameter κ.

In equations (4.1) (see [8], § 95) g_{ik} is the metric tensor, $\mathcal{R}_{ik}$ is the Ricci tensor, $\mathcal{R}$ is a trace of the Ricci tensor.

It follows from the Einstein equations (4.1) that, where the values of the components of the tensor T_{ik} are of the order of unity ($T_{ik} = O(1)$), the components of the Ricci tensor satisfy the estimate $\mathcal{R}_{ik} = O(\kappa^2)$. However, when calculating the components of the energy-momentum pseudo-tensor of the gravitational field t_{ik} (see [8], § 96), it is easy to verify that $t_{ik} = \frac{1}{2Gr}\Theta_{ik}$, where the components of the very cumbersome pseudo-tensor Θ_{ik}, constructed from the metric tensor and the Christoffel symbol, have the same order of magnitude as the Ricci tensor. Consequently, the components of the pseudo-tensor t_{ik} seem to "forget" about the smallness of the gravitational constant Gr and have the same order as the components of the tensor T_{ik}: the energy of the gravitational coupling has the same order of magnitude as the electromagnetic energy.

In units of e, c, r_0, the mass unit m_0 is the value $m_0 = \frac{e^2}{r_0 c^2}$. This value is of the order of 10^{-13} g. In these units, the Stoney's mass m_s is defined by the expression $m_s = \frac{1}{\kappa}$. Accordingly, the above considerations suggest that the observed electron mass m in units of m_0 satisfies the estimate $m = O(\kappa^2)$.

Constructing the existence theorem for the theory [1], it is necessary to establish the existence conditions for the solution of the stationary single-particle problem. In particular, it is necessary to establish: how many normalization conditions can be used? For what values of the parameter κ do exist solutions? (It seems plausible to assume that solutions exist only for sufficiently small κ). In addition, it is necessary to prove that $m > 0$ (the gravitational coupling energy cannot exceed the particle's electromagnetic energy), and also to prove (or disprove) the ordinal estimation for m given above: $m = O(\kappa^2)$. It is also necessary to establish whether the angular momentum of a classical lepton is uniquely determined for a given topology of the particle boundaries.

It would be rash to expect that the classical electrodynamics of non-point charges [1] can accurately predict the parameters of leptons in full accordance with experience. The goal of developing a classical theory of this type (as Paul Dirac repeatedly insisted) is to be a good basis for the development of a quantum relativistic theory free from regularization and renormalization. A sketch of such a quantum theory is contained in the author's article [9].

In essence, the classical relativistic theory of leptons [1], like the non-relativistic Bohr's theory of the atom, can, in certain situations, correspond to experience only when some additional "quasi-Bohr's" conditions are imposed on the solution. One such "quasi-Bohr's" condition must be imposed when normalizing to a unit of a full charge of a lepton: staying within the framework of classical physics, we do not know why all massive leptons have the same electric charge. We simply use this fact to normalize the solutions.

Without having a theorem on the existence of a solution, we do not know whether we have the right to set another "quasi-Bohr's" condition, normalizing own angular momentum M to its empirical value: $M = \frac{1}{2\alpha}$ (in units of angular momentum $\frac{e^2}{c}$), where $\alpha = \frac{e^2}{\hbar c}$ is the fine-structure constant. If, to construct a solution, we have the right to use only one normalization condition (the "quasi-Bohr's" condition for charge normalization), then we must simply humbly calculate own angular momentum M within the framework of the theory [1] and compare it with the experimental value: this will be a calculation of the quantum constant Planck $\hbar$ in the framework of the classical theory!

Additional difficulties in this problem are caused by the pseudo-tensor nature of t_{ik}: local values of these quantities (which must be calculated with an accuracy of about 25-30 significant digits!) are generally not physically meaningful and can be arbitrarily changed by a simple coordinate transformation. Only integrals over the entire three-dimensional space have physical meaning. Those researchers who venture to tackle the question of the existence of stationary solutions of the theory [1] may first consider a simpler model problem of the lepton structure in a world without gravity. This is a flat world with the geometry of Minkowski and without the energy of a gravitational coupling. Such a world without gravity cannot be compared with the world in which we live, but the proof of the existence theorem and the numerical solution of the problem will undoubtedly be simpler (there is no small parameter κ, which is quite "bad" in the theory [1]: it is in the numerator in some equations and in the denominator in other expressions).

Probably, it would be of some interest to study the model problem with the opposite nature and with the apparent absence of matter – that is, with full local "devouring" of electromagnetic energy by the energy of gravitational coupling. Within the framework of such a model, the electrodynamics equations are not taken into account, and in the right-hand part of the Einstein equations the tensor T_{ik} is replaced by – t_{ik}. This model does not contain a constant of gravity. This model is questionable from the point of view of mathematics (a tensor is equated to a pseudo-tensor), but if the solution to such a problem exists in a certain coordinate system, then it describes a certain "geometric trap" that does not contain matter.

H. Kleinert considered a model in some sense related to such a geometric trap [10]. He studied the singular solutions of homogeneous Einstein equations as a model of dark matter.

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References

1. *Temnenko, V. A.* Physics of current and potentials. I. Classical electrodynamics with non-point charge // Electronic Journal of Theoretical Physics. — 2014. — Vol. 11, No. 31. — P. 221–256.

2. *Temnenko, V. A.* Physics of current and potentials. III. Octuplet sector of classical field theory with non-point particles // Electronic Journal of Theoretical Physics. — 2016. — Vol. 13, No. 36. — P. 69–98.
3. *Ray, T. R.* Stoney's fundamental units // Irish Astronomical Journal. — 1981. — Vol. 15. — P. 152.
4. *Weinberg, S.* The discovery of subatomic participles. Revised edition. — Cambridge, Cambridge University Press, 2003. — 206 p.
5. *Temnenko, V. A.* Physics of current and potentials. II. Classical singlet-triplet electroweak theory with non-point particle // Electronic Journal of Theoretical Physics. — 2015. — Vol. 12, No. 32. — P. 179–294.
6. *F*ine-tuning [Electronic resource]. — https://bit.ly/2SgprgD, 25.12.2018.
7. *H*ierarchy problem [Electronic resource]. — https://bit.ly/1jkEtAv, 25.12.2018.
8. *Landau, L. D., Lifshitz, E. M.* The classical theory of fields. – Fourth rev. eng. edition. — Elsevier Ltd, Amsterdam e.a., 1975. — 428 p.
9. *Temnenko, V. A.* Physics of current and potentials. IV. Dirac space and Dirac vectors in the quantum relativistic theory // Electronic Journal of Theoretical Physics. — 2018. — Vol. 14, No. 37. — P. 213–249.
10. *Kleinert, H.* The GIMP Nature of dark matter // Electronic Journal of Theoretical Physics. — 2016. — Vol. 13, No. 36. — P. 1–12.

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