Динамические системы, 2015, том 5(33), №1-2, 3–11

MSC 2010: 34K20, 34K40

Exponential stability of solutions to one class of nonlinear systems of neutral type with periodic coefficients in linear terms[1]

G. V. Demidenko, I. I. Matveeva
Sobolev Institute of Mathematics, Novosibirsk 630090, Russia.
Novosibirsk State University, Novosibirsk 630090, Russia.
E-mail: demidenk@math.nsc.ru, matveeva@math.nsc.ru

Abstracts. We study the problem on exponential stability of the zero solution to a class of nonlinear systems of neutral type with periodic coefficients in linear terms. We establish conditions of exponential stability and obtain estimates for solutions.

Keywords: time-delay systems, neutral type, periodic coefficients, exponential stability, Lyapunov–Krasovskii functional.

1. Introduction

Problems on stability of solutions to autonomous delay differential equations are studied well (for instance, see [1, 2, 11, 12, 15, 16, 17, 18, 19, 21] and the bibliography therein). However, there are much less results on the stability theory for nonautonomous delay equations. The main investigations for nonautonomous equations focus on linear delay differential equations with periodic coefficients

$$\frac{d}{dt}y(t) = A(t)y(t) + B(t)y(t-\tau), \quad A(t+T) \equiv A(t), \quad B(t+T) \equiv B(t), \quad t > 0. \tag{1.1}$$

The fundamentals of the stability theory of solutions to these equations were laid in the papers [13, 14, 22, 23] and others. One of the main approaches when studying stability of solutions to (1.1) is the development of the Floquet theory and the use of the monodromy operator. This approach is also applied when studying stability of solutions to linear equations of neutral type with periodic coefficients

$$\frac{d}{dt}\left(y(t) + Dy(t-\tau)\right) = A(t)y(t) + B(t)y(t-\tau), \quad t > 0. \tag{1.2}$$

To study asymptotic stability of the zero solution to the time-delay systems (1.1) and (1.2), the authors in [5, 6, 7, 9] proposed to use Lyapunov–Krasovskii functionals of the form

$$V(t,y) = \langle H(t)(y(t) + Dy(t-\tau)), (y(t) + Dy(t-\tau))\rangle + \int_{t-\tau}^{t} \langle K(t-s)y(s), y(s)\rangle\, ds. \tag{1.3}$$

In particular, the following result was established in [9].

[1] The work is supported in part by the Russian Foundation for Basic Research (project no. 13-01-00329).

© *G. V. DEMIDENKO, I. I. MATVEEVA*

Theorem 1. *Suppose that there exist matrices* $H(t) = H^*(t) \in C^1[0,T]$, $K(s) = K^*(s) \in C^1[0,\tau]$ *such that*

$$H(0) = H(T) > 0, \quad K(s) > 0, \quad \frac{d}{ds}K(s) < 0, \quad s \in [0,\tau],$$

and the matrix

$$C(t) = \begin{pmatrix} C_{11}(t) & C_{12}(t) \\ C_{12}^*(t) & C_{22}(t) \end{pmatrix} \tag{1.4}$$

with

$$C_{11}(t) = -\frac{d}{dt}H(t) - H(t)A(t) - A^*(t)H(t) - K(0),$$

$$C_{12}(t) = -\frac{d}{dt}H(t)D - H(t)B(t) - A^*(t)H(t)D,$$

$$C_{22}(t) = -D^*\frac{d}{dt}H(t)D - D^*H(t)B(t) - B^*(t)H(t)D + K(\tau)$$

is positive definite for $t \in [0,T]$*. Then the zero solution to* (1.2) *is exponentially stable.*

Using functionals of the form (1.3), the authors in [5, 6, 9] obtained first analogues of M. G. Krein's estimates [3] characterizing exponential decay of solutions to (1.1) and (1.2) as $t \to \infty$. In the case of ordinary differential equations with periodic coefficients ($D \equiv B(t) \equiv 0$), first estimates of such kind were obtained in [4].

In the present paper we use the Lyapunov–Krasovskii functional (1.3) in order to study stability of the zero solution to the system of nonlinear delay differential equations

$$\frac{d}{dt}\left(y(t) + Dy(t-\tau)\right) = A(t)y(t) + B(t)y(t-\tau) + F(t, y(t), y(t-\tau)), \qquad t > 0, \tag{1.5}$$

where D is a constant $(n \times n)$-matrix, $A(t)$, $B(t)$ are $(n \times n)$-matrices with continuous T-periodic entries, $\tau > 0$ is the time delay, and $F(t,u,v)$ is a continuous vector function satisfying the inequality

$$\|F(t,u,v)\| \le q_1\|u\| + q_2\|v\|, \quad q_1,\ q_2 \ge 0 \text{ are constant.} \tag{1.6}$$

Our aim is to establish conditions of exponential stability and to obtain estimates characterizing exponential decay of solutions to (1.5) at infinity. This paper continues our investigations of nonlinear time-delay systems [8, 10, 20]. Some examples illustrating effectiveness of our approach are given in [10].

2. Main results

Supposing that the conditions of Theorem 1 are satisfied, we formulate our main results in this section. Using the matrices $H(t)$ and $K(s)$, introduce the following notation

$$S(t) = \begin{pmatrix} S_{11}(t) & S_{12}(t) \\ S_{12}^*(t) & S_{22}(t) \end{pmatrix}, \tag{2.1}$$

$$S_{11}(t) = -\frac{d}{dt}H(t) - H(t)A(t) - A^*(t)H(t) - K(0),$$

$$S_{12}(t) = H(t)A(t)D + K(0)D(t) - H(t)B(t), \quad S_{22}(t) = K(\tau) - D^*K(0)D,$$
$$q(t) = \left(q_1 + \sqrt{q_1^2 + (q_1\|D\| + q_2)^2}\right)\|H(t)\|. \tag{2.2}$$

Here and thereafter we use the spectral norm of matrices. It is not hard to verify that $S(t)$ is positive definite if and only if $C(t)$ in (1.4) is positive definite (see Section 3 for details). Denote by I the unit matrix.

Theorem 2. *Let the conditions of Theorem 1 be satisfied. Suppose that q_1, q_2 are such that the matrix $(S(t) - q(t)I)$ is positive definite for $t \in [0,T]$. Then the zero solution to (1.5) is exponentially stable.*

Consider the initial value problem for (1.5)

$$\begin{aligned}
&\frac{d}{dt}(y(t) + Dy(t-\tau)) = A(t)y(t) + B(t)y(t-\tau) + F(t, y(t), y(t-\tau)), \quad t > 0,\\
&y(t) = \varphi(t), \quad t \in [-\tau, 0],\\
&y(+0) = \varphi(0),
\end{aligned} \tag{2.3}$$

where $\varphi(t) \in C^1[-\tau, 0]$ is a given vector function. Below we establish estimates of solutions to the initial value problem (2.3) characterizing the rate of exponential decay as $t \to \infty$.

To formulate our results, we introduce some notations. If the matrix $H(t)$ satisfies the conditions of Theorem 1 then

$$\frac{d}{dt}H(t) + H(t)A(t) + A^*(t)H(t) < -K(0).$$

It follows from the authors' results in [4] that $H(t) > 0$ on $[0,T]$. We extend T-periodically the matrix $H(t)$ to the whole half-axis $\{t > 0\}$, keeping the same notation. Using this matrix $H(t)$ and the matrix $K(s)$, satisfying the conditions of Theorem 1, we consider the functional (1.3) and put

$$V_0(\varphi) = \langle H(0)(\varphi(0) + D\varphi(-\tau)), (\varphi(0) + D\varphi(-\tau))\rangle + \int_{-\tau}^{0} \langle K(-s)\varphi(s), \varphi(s)\rangle ds. \tag{2.4}$$

We introduce

$$\begin{aligned}
P(t) = -\frac{d}{dt}H(t) - H(t)A(t) - A^*(t)H(t) - K(0) - q(t)I\\
- (H(t)A(t)D + K(0)D - H(t)B(t))[K(\tau) - D^*K(0)D - q(t)I]^{-1}\\
\times (H(t)A(t)D + K(0)D - H(t)B(t))^*.
\end{aligned} \tag{2.5}$$

Note that $P(t)$ is positive definite if the matrix $(S(t) - q(t)I)$ is positive definite (see Section 3 for details). Denote by $p_{\min}(t) > 0$ the minimal eigenvalue of $P(t)$ and by $h_{\min}(t) > 0$ the minimal eigenvalue of $H(t)$. Let $k > 0$ be the maximal number such that

$$\frac{d}{ds}K(s) + kK(s) \le 0, \quad s \in [0, \tau]. \tag{2.6}$$

We put

$$\gamma(t) = \min\{p_{\min}(t),\ k\|H(t)\|\}, \tag{2.7}$$

$$\begin{gathered}\Phi = \max_{t\in[-\tau,0]}\|\varphi(t)\|, \quad \alpha = \max_{t\in[0,T]}\sqrt{\frac{V_0(\varphi)}{h_{\min}(t)}},\\ \beta(t) = \frac{\gamma(t)}{2\|H(t)\|}, \quad \beta^+ = \max_{t\in[0,T]}\beta(t), \quad \beta^- = \min_{t\in[0,T]}\beta(t).\end{gathered} \tag{2.8}$$

Analogy as in [9], we distinguish three cases when obtaining estimates. It is not hard to show that the spectrum of the matrix D belongs to the unit disk $\{\lambda\in\mathbb{C} : |\lambda| < 1\}$ if the conditions of Theorem 1 are fulfilled; i.e., if the matrix $C(t)$ (or $S(t)$) is positive definite. Consequently, $\|D^j\| \to 0$ as $j\to\infty$. Let $l > 0$ be the minimal integer such that $\|D^l\| < 1$.

Theorem 3. *Let the conditions of Theorem 2 be satisfied.*

I. *If* $\|D^l\| < e^{-l\beta^+\tau}$ *then a solution to the initial value problem* (2.3) *satisfies the estimate*

$$\|y(t)\| \le \Bigg[\alpha\big(1-\|D^l\|e^{l\beta^+\tau}\big)^{-1}\sum_{j=0}^{l-1}\|D^j\|e^{j\beta^+\tau} + \max\Big\{\|D\|e^{\beta^+\tau},\dots,\|D^l\|e^{l\beta^+\tau}\Big\}\,\Phi\Bigg]e^{-\int_0^t\beta(\xi)\,d\xi}, \qquad t>0.$$

II. *If* $e^{-l\beta^+\tau} \le \|D^l\| \le e^{-l\beta^-\tau}$ *then a solution to the initial value problem* (2.3) *satisfies the estimate*

$$\|y(t)\| \le \Bigg[\alpha\left(1+\frac{t}{l\tau}\right)\sum_{j=0}^{l-1}\|D^j\|e^{j\beta^+\tau} + \max\Big\{1,\|D\|e^{\beta^+\tau},\dots,\|D^{l-1}\|e^{(l-1)\beta^+\tau}\Big\}\,\Phi\Bigg]e^{-\int_0^t\widetilde\beta(\xi)\,d\xi}, \qquad t>0.$$

III. *If* $e^{-l\beta^-\tau} < \|D^l\| < 1$ *then a solution to the initial value problem* (2.3) *satisfies the estimate*

$$\|y(t)\| \le \Bigg[\alpha\|D^l\|e^{l\beta^-\tau}\Big(\|D^l\|e^{l\beta^-\tau}-1\Big)^{-1}\sum_{j=0}^{l-1}\|D^j\|e^{j\beta^-\tau} + \|D^l\|^{\frac{1}{l}-1}\max\Big\{1,\|D\|,\dots,\|D^{l-1}\|\Big\}\,\Phi\Bigg]\exp\left(\frac{t}{l\tau}\ln\|D^l\|\right), \qquad t>0.$$

Here α, $\beta(t)$, β^+, β^-, *and* Φ *are defined in* (2.8), $\widetilde\beta(t) = \min\left\{\beta(t),\ -\frac{1}{l\tau}\ln\|D^l\|\right\} > 0$.

We prove Theorem 3 in Section 3. Obviously, Theorem 2 immediately follows from this theorem.

3. Proof of the main results

To prove Theorem 3 we need auxiliary results obtained below.

Lemma 1. *Let the conditions of Theorem 2 be satisfied. Then, for a solution to the initial value problem* (2.3)*, the following inequality holds*

$$\|y(t)+Dy(t-\tau)\| \le \sqrt{\frac{V_0(\varphi)}{h_{\min}(t)}} \exp\left(-\int\limits_0^t \frac{\gamma(\xi)}{2\|H(\xi)\|}d\xi\right), \qquad t>0, \tag{3.1}$$

where $V_0(\varphi)$ *and* $\gamma(t)$ *are defined by* (2.4) *and* (2.7)*, respectively,* $h_{\min}(t)>0$ *is the minimal eigenvalue of the matrix* $H(t)$*.*

Proof. We follow the strategy in [5]. Let $y(t)$ be a solution to the initial value problem (2.3). Using the above matrices $H(t)$ and $K(s)$, we consider the Lyapunov–Krasovskii functional (1.3) on this solution. Using the matrix $C(t)$ defined in (1.4), the time derivative of this functional can be written as follows

$$\begin{aligned}\frac{d}{dt}V(t,y) \equiv -\left\langle C(t)\begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix}, \begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix}\right\rangle \\ + \langle H(t)F(t,y(t),y(t-\tau)),(y(t)+Dy(t-\tau))\rangle \\ + \langle H(t)(y(t)+Dy(t-\tau)),F(t,y(t),y(t-\tau))\rangle \\ + \int\limits_{t-\tau}^{t}\left\langle \frac{d}{dt}K(t-s)y(s),y(s)\right\rangle ds.\end{aligned} \tag{3.2}$$

Consider the first summand in the right-hand side of (3.2). Since

$$\begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix} = \begin{pmatrix} I & -D\\ 0 & I\end{pmatrix}\begin{pmatrix} y(t)+Dy(t-\tau)\\ y(t-\tau)\end{pmatrix}$$

then

$$\left\langle C(t)\begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix}, \begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix}\right\rangle \equiv \left\langle S(t)\begin{pmatrix} y(t)+Dy(t-\tau)\\ y(t-\tau)\end{pmatrix}, \begin{pmatrix} y(t)+Dy(t-\tau)\\ y(t-\tau)\end{pmatrix}\right\rangle,$$

where the matrix $S(t)$ is defined in (2.1). Obviously, if the matrix $C(t)$ is positive definite then the matrix $S(t)$ is positive definite as well.

Now we consider the second and the third summands in the right-hand side of (3.2). In view of (1.6) we have

$$\begin{aligned}&\langle H(t)F(t,y(t),y(t-\tau)),(y(t)+Dy(t-\tau))\rangle + \langle H(t)(y(t)+Dy(t-\tau)),F(t,y(t),y(t-\tau))\rangle\\ &\le 2\|H(t)\|\big(q_1\|y(t)\|+q_2\|y(t-\tau)\|\big)\|y(t)+Dy(t-\tau)\|\\ &\le 2q_1\|H(t)\|\|y(t)+Dy(t-\tau)\|^2 + 2(q_1\|D\|+q_2)\|H\|\|y(t-\tau)\|\|y(t)+Dy(t-\tau)\|\\ &\le q(t)\big(\|y(t)+Dy(t-\tau)\|^2+\|y(t-\tau)\|^2\big),\end{aligned}$$

where $q(t)$ is given in (2.2).

Hence,

$$\begin{aligned}&-\left\langle C(t)\begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix}, \begin{pmatrix} y(t)\\ y(t-\tau)\end{pmatrix}\right\rangle\\ &+\langle H(t)F(t,y(t),y(t-\tau)),(y(t)+Dy(t-\tau))\rangle + \langle H(t)(y(t)+Dy(t-\tau)),F(t,y(t),y(t-\tau))\rangle\\ &\le -\left\langle (S(t)-q(t)I)\begin{pmatrix} y(t)+Dy(t-\tau)\\ y(t-\tau)\end{pmatrix}, \begin{pmatrix} y(t)+Dy(t-\tau)\\ y(t-\tau)\end{pmatrix}\right\rangle.\end{aligned} \tag{3.3}$$

By the conditions of Theorem 2, the matrix $(S(t) - q(t)I)$ is positive definite. Using the representation

$$S(t) - q(t)I = \begin{pmatrix} I & S_{12}(t)(S_{22}(t) - q(t)I)^{-1} \\ 0 & I \end{pmatrix}$$
$$\times \begin{pmatrix} S_{11}(t) - q(t)I - S_{12}(t)(S_{22}(t) - q(t)I)^{-1}S_{12}^*(t) & 0 \\ 0 & S_{22}(t) - q(t)I \end{pmatrix}$$
$$\times \begin{pmatrix} I & 0 \\ (S_{22}(t) - q(t)I)^{-1}S_{12}^*(t) & I \end{pmatrix},$$

we have

$$\left\langle (S(t) - q(t)I)\begin{pmatrix} y(t) + Dy(t-\tau) \\ y(t-\tau) \end{pmatrix}, \begin{pmatrix} y(t) + Dy(t-\tau) \\ y(t-\tau) \end{pmatrix} \right\rangle$$
$$\geq \left\langle \left[S_{11}(t) - q(t)I - S_{12}(t)(S_{22}(t) - q(t)I)^{-1}S_{12}^*(t)\right](y(t) + Dy(t-\tau)), (y(t) + Dy(t-\tau)) \right\rangle.$$

Since the matrix $(S(t) - q(t)I)$ is positive definite then the matrix

$$P(t) = S_{11}(t) - q(t)I - S_{12}(t)(S_{22}(t) - q(t)I)^{-1}S_{12}^*(t)$$

is positive definite. Taking into account (2.1), the matrix $P(t)$ has the form (2.5). Consequently, from (3.3) we derive

$$\begin{aligned} &- \left\langle C(t)\begin{pmatrix} y(t) \\ y(t-\tau) \end{pmatrix}, \begin{pmatrix} y(t) \\ y(t-\tau) \end{pmatrix} \right\rangle \\ &\qquad + \langle H(t)F(t, y(t), y(t-\tau)), (y(t) + Dy(t-\tau)) \rangle \\ &\qquad + \langle H(t)(y(t) + Dy(t-\tau)), F(t, y(t), y(t-\tau)) \rangle \\ &\qquad \leq - \langle P(t)(y(t) + Dy(t-\tau)), (y(t) + Dy(t-\tau)) \rangle \\ &\qquad \leq -p_{\min}(t)\|y(t) + Dy(t-\tau)\|^2, \end{aligned} \quad (3.4)$$

where $p_{\min}(t) > 0$ is the minimal eigenvalue of $P(t)$. Using the matrix $H(t)$, we have

$$\begin{aligned} h_{\min}(t)\,\|y(t) + Dy(t-\tau)\|^2 &\leq \langle H(t)(y(t) + Dy(t-\tau)), (y(t) + Dy(t-\tau)) \rangle \\ &\leq \|H(t)\|\,\|y(t) + Dy(t-\tau)\|^2, \end{aligned}$$

where $h_{\min}(t) > 0$ is the minimal eigenvalue of $H(t)$. Using (2.6) and (3.4), from (3.2) we derive

$$\begin{aligned} \frac{d}{dt}V(t,y) \leq -\frac{p_{\min}(t)}{\|H(t)\|}\,&\langle H(t)(y(t) + Dy(t-\tau)), (y(t) + Dy(t-\tau)) \rangle \\ &- k\int_{t-\tau}^{t} \langle K(t-s)y(s), y(s) \rangle\, ds. \end{aligned}$$

Taking into account the definition of the functional (1.3), we obtain

$$\frac{d}{dt}V(t,y) \leq -\frac{\gamma(t)}{\|H(t)\|}V(t,y),$$

where $\gamma(t)$ is defined by (2.7). From this differential inequality we derive the estimate

$$V(t,y) \le V_0(\varphi)\exp\left(-\int_0^t \frac{\gamma(\xi)}{\|H(\xi)\|}d\xi\right).$$

Then, using the definition of the functional (1.3), we have

$$\|y(t)+Dy(t-\tau)\| \le \sqrt{\frac{V(t,y)}{h_{\min}(t)}} \le \sqrt{\frac{V_0(\varphi)}{h_{\min}(t)}}\exp\left(-\int_0^t \frac{\gamma(\xi)}{2\|H(\xi)\|}d\xi\right).$$

The lemma is proved. □

Lemma 2. *Let the conditions of Theorem 2 be satisfied. Then a solution to the initial value problem* (2.3) *on every segment* $t \in [k\tau,(k+1)\tau)$, $k=0,1,\dots,$ *satisfies the following estimate*

$$\|y(t)\| \le \alpha \sum_{j=0}^{k} \|D^j\| e^{-\int_0^{t-j\tau}\beta(\xi)\,d\xi} + \|D^{k+1}\|\Phi, \tag{3.5}$$

where α, $\beta(t)$, *and* Φ *are defined in* (2.8).

Proof. By Lemma 1, a solution to the initial value problem (2.3) satisfies (3.1). In view of the notations (2.8), the estimate has the form

$$\|y(t)+Dy(t-\tau)\| \le \alpha e^{-\int_0^t \beta(\xi)\,d\xi}, \qquad t>0. \tag{3.6}$$

Obviously, for $t \in [0,\tau)$ we have the inequality

$$\|y(t)\| \le \alpha e^{-\int_0^t \beta(\xi)\,d\xi} + \|Dy(t-\tau)\| \le \alpha e^{-\int_0^t \beta(\xi)\,d\xi} + \|D\|\Phi,$$

which gives us (3.5) for $k=0$.

Let $t \in [k\tau,(k+1)\tau)$, $k=1,2\dots$. It is not hard to write out the sequence of the inequalities

$$\begin{aligned}
\|y(t)\| &\le \alpha e^{-\int_0^t \beta(\xi)\,d\xi} + \|Dy(t-\tau)\| \\
&\le \alpha e^{-\int_0^t \beta(\xi)\,d\xi} + \|Dy(t-\tau)+D^2y(t-2\tau)\| + \|D^2y(t-2\tau)+D^3y(t-3\tau)\| + \dots \\
&\qquad + \|D^ky(t-k\tau)+D^{k+1}y(t-(k+1)\tau)\| + \|D^{k+1}y(t-(k+1)\tau)\| \\
&\le \alpha e^{-\int_0^t \beta(\xi)\,d\xi} + \|D\|\,\|y(t-\tau)+Dy(t-2\tau)\| + \|D^2\|\,\|y(t-2\tau)+Dy(t-3\tau)\| + \dots \\
&\qquad + \|D^k\|\,\|y(t-k\tau)+Dy(t-(k+1)\tau)\| + \|D^{k+1}\|\,\|y(t-(k+1)\tau)\|.
\end{aligned}$$

By (3.6) we derive the estimate

$$\begin{aligned}
\|y(t)\| \le \alpha e^{-\int_0^t \beta(\xi)\,d\xi} &+ \alpha\|D\|e^{-\int_0^{t-\tau} \beta(\xi)\,d\xi} + \alpha\|D^2\|e^{-\int_0^{t-2\tau} \beta(\xi)\,d\xi} + \dots \\
&+ \alpha\|D^k\|e^{-\int_0^{t-k\tau} \beta(\xi)\,d\xi} + \|D^{k+1}\|\Phi,
\end{aligned}$$

which implies (3.5).

The lemma is proved. □

Proof of Theorem 3. For the linear time-delay system (1.2) with periodic coefficients, the analogues of Theorem 3 (see Theorems 2–4 in [9]) were proved in detail in [9] by the use of the auxiliary assertions (see Lemmas 2–4 in [9]). In the present paper, using Lemmas 1, 2 and repeating the reasoning carried out when proving Theorems 2–4 in [9], we derive the required estimates for solutions to the initial value problem (2.3). □

Using the proof of Theorem 2, we can reformulate the conditions of exponential stability of the zero solution to the nonlinear system (1.5) as follows.

Theorem 4. *Suppose that there exist matrices*

$$H(t) = H^*(t) \in C^1[0,T], \qquad H(0) = H(T) > 0,$$

$$K(s) = K^*(s) \in C^1[0,\tau], \quad K(s) > 0, \quad \frac{d}{ds}K(s) < 0, \quad s \in [0,\tau],$$

such that the matrices $(K(\tau) - D^*K(0)D - q(t)I)$ *and* $P(t)$ *defined by* (2.5) *are positive definite for* $t \in [0,T]$. *Then the zero solution to* (1.5) *is exponentially stable.*

Remark 1. Taking into account the form of $P(t)$, we see that the conditions of exponential stability can be formulated in terms of the linear matrix inequality and the differential matrix inequality of Riccati type.

Remark 2. Theorem 3 makes it possible to estimate the rate of exponential decay of solutions to (2.3) at infinity; moreover, all the values characterizing the decay rate are obtained in explicit form.

References

1. *Agarwal R. P., Berezansky L., Braverman E., Domoshnitsky A.* Nonoscillation theory of functional differential equations with applications. — New York: Springer, 2012.
2. *Azbelev N. V., Maksimov V. P., Rakhmatullina L. F.* Introduction to the theory of functional differential equations: methods and applications. — Vol. 3: Contemporary mathematics and its applications. — Cairo: Hindawi Publishing Corporation, 2007.
3. *Daleckii Ju. L., Krein M. G.* Stability of solutions of differential equations in Banach space. — Vol. 43: Translations of mathematical monographs. — Providence: Amer. Math. Soc., 1974.
4. *Demidenko G. V., Matveeva I. I.* On stability of solutions to linear systems with periodic coefficients // Sib. Math. J. — 2001. — Vol. 42, no. 2. — P. 282–296.
5. *Demidenko G. V., Matveeva I. I.* Asymptotic properties of solution to delay differential equations // Vestnik Novosib. Gos. Univ. Ser. Mat. Mekh. Inform. — 2005. — Vol. 5, no. 3. — P. 20–28.
6. *Demidenko G. V., Matveeva I. I.* Stability of solutions to delay differential equations with periodic coefficients of linear terms // Sib. Math. J. — 2007. — Vol. 48, no. 5. — P. 824–836.
7. *Demidenko G. V.* Stability of solutions to linear differential equations of neutral type // J. Anal. Appl. — 2009. — Vol. 7, no. 3. — P. 119–130.
8. *Demidenko G. V., Kotova T. V., Skvortsova M. A.* Stability of solutions to differential equations of neutral type // J. Math. Sci. — 2012. — Vol. 186, no. 3. — P. 394–406.
9. *Demidenko G. V., Matveeva I. I.* On estimates of solutions to systems of differential equations of neutral type with periodic coefficients // Sib. Math. J. — 2014. — Vol. 55, no. 5. — P. 866–881.

10. *Demidenko G. V., Matveeva I. I.* Estimates for solutions to a class of nonlinear time-delay systems of neutral type // Electron. J. Diff. Equ. — 2015. — Vol. 2015, no. 34. — P. 1–14.
11. *Gu K., Kharitonov V. L., Chen J.* Stability of time-delay systems. — Control engineering. — Boston: Birkhäuser, 2003.
12. *Györi I., Ladas G.* Oscillation theory of delay differential equations. With applications. — Oxford mathematical monographs. Oxford science publications. — New York: The Clarendon Press, Oxford University Press, 1991.
13. *Hahn W.* On difference differential equations with periodic coefficients // J. Math. Anal. Appl. — 1961. — Vol. 3, no. 1. — P. 70–101.
14. *Halanay A.* Stability theory of linear periodic systems with retardation // Acad. Républ. Popul. Roum., Rev. Math. Pures Appl. — 1961. — Vol. 6, no. 4. — P. 633–653.
15. *Hale J. K.* Theory of Functional Differential Equations. — New York, Heidelberg, Berlin: Springer-Verlag, 1977.
16. *Kharitonov V. L.* Time-delay systems. Lyapunov functionals and matrices. — Control engineering. — New York: Birkhäuser, Springer, 2013.
17. *Khusainov D. Ya., Shatyrko A. V.* The method of Lyapunov functions in the investigation of the stability of functional differential systems. (Russian) — Kiev: Kiev University Press, 1997.
18. *Kolmanovskii V. B., Myshkis A. D.* Introduction to the theory and applications of functional differential equations. — Vol. 463: Mathematics and its applications. — Dordrecht: Kluwer Academic Publishers, 1999.
19. *Korenevskii D. G.* Stability of dynamical systems under random perturbations of parameters. Algebraic criteria. (Russian) — Kiev: Naukova Dumka, 1989.
20. *Matveeva I. I.* Estimates of solutions to a class of systems of nonlinear delay differential equations // J. Appl. Indust. Math. — 2013. — Vol. 7, no. 4. — P. 557–566.
21. *Michiels W., Niculescu S. I.* Stability and stabilization of time-delay systems. An eigenvalue-based approach. — Vol. 12: Advances in design and control. — Philadelphia: Society for Industrial and Applied Mathematics, 2007.
22. *Stokes A. P.* A Floquet theory for functional differential equations // Proc. Nat. Acad. Sci. USA. — 1962. — Vol. 48, no. 8. — P. 1330–1334.
23. *Zverkin A. M.* On the theory of linear differential equations with a lagging argument and periodic coefficients // Dokl. Akad. Nauk SSSR. — 1959. — Vol. 128, no. 5. — P. 882–885.

Получена 03.06.2015